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AN INVESTIGATION OF ENERGY SEPARATION IN A VORTEX TUBE

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Abstract—The process of energy separation in a vortex tube with air as a working medium is studied in detail. Experimental data of the temperatures of the cold and hot air leaving the vortex tube are presented. The variation of the maximum wall temperature along the vortex tube surface provides useful information about the location of the stagnation point of the flow field at the axis of the vortex tube. Experimental results indicate that the Görtler vortex produced by the tangential velocity on the inside wall of the vortex tube is a major driving force for the energy separation. A similarity relation for the prediction of the temperature of the cold exit air, obtained from the dimensional analysis, is presented and confirmed by experimental data.

NOMENCLATURE

C^*,	dimensionless parameter;
c_p,	specific heat at constant pressure;
c_v,	specific heat at constant volume;
D,	inner diameter of vortex tube;
d_c, d_h,	diameter of cold and hot gas exit;
D_c, D_h,	dimensionless diameter of cold and hot gas exit;
Eu,	Euler number;
k,	thermal conductivity;
L, L^*,	length and dimensionless length of vortex tube;
$\dot{m}$,	mass flow rate;
Ma,	Mach number;
p, p^*,	pressure and dimensionless pressure;
p_∞,	ambient pressure;
Pr,	Prandtl number;
r, θ, z,	cylindrical coordinates;
Re,	Reynolds number;
T, θ,	temperature and dimensionless temperature;
ΔT,	temperature difference;
w_0,	inlet velocity;
y,	cold gas mass ratio.

Greek symbols

β, β^*,	coefficient of thermal expansion and dimensionless coefficient of thermal expansion;
γ,	ratio of the specific heats, c_p/c_v;
δ, Δ,	width and dimensionless width of the inlet nozzle of the vortex tube;
μ,	dynamic viscosity;
ρ,	density.

Subscripts

c,	cold gas;
h,	hot gas;
o,	inlet;
w,	wall.

1. INTRODUCTION

THE PHENOMENON of energy separation in a vortex tube was first reported by Ranque [1]. He investigated compressed air flowing tangentially into a tube and found that air leaving the tube near the wall had a higher temperature than that leaving near the axis. He tried to use this effect for refrigeration. Hilsch [2] first published systematical experimental results by varying the inlet pressure and the geometrical parameters of the tube. Comparing the vortex tube to conventional refrigerating machines, the former has a much lower coefficient of performance than the latter, especially at low inlet pressures. Nevertheless, he suggested that the vortex tube could be used for the liquefaction of natural gas. In the following years, attempts were made to explain the effect of the energy separation in the vortex tube. Fulton [3] assumed that the effect results from the exchange of energy between the air near the axis, which has a higher angular velocity, and the air at the periphery, which has a low angular velocity: the air near the axis tries to accelerate the exterior air. Besides a mathematical formulation of the problem, Fulton also suggested a shape for the flow pattern inside the tube with radial and axial flow components. A different theory was developed by Schultz-Grunow [4]. He explained the energy separation mainly by turbulent heat transfer in a stratified flow. Martynovskii and Alekseev [5] investigated in an experimental study the influence of different geometrical constructions of the vortex tube on the maximum possible temperature difference between the cold and hot gas streams. They used various inlet nozzles for geometrical optimization. Experiments were conducted with ammonia, methane and carbon dioxide. They concluded from their theoretical consideration that, for turbulent Prandtl numbers $Pr^* < 0.5$, a reversal effect should occur in the vortex tube. In this case, the cold gas leaves the tube at the periphery and the hot gas near the axis. Hartnett and Eckert [6] measured profiles of the pressure and temperature at different positions in a vortex tube. They found that the pressure and temperature vary greatly along the axis of the vortex tube. This led them to the conclusion that the length of the vortex tube has an important influence on the mechanism of the energy separation. In a theoretical study Deissler and Perlmutter [7] made an analysis of the velocity, temperature and pressure distributions in a turbulent vortex with radial and axial flow. In their mathematical model they divided the vortex into a core and an annular region. They concluded that the fluid in the core region does shear work on the fluid in the outer region with a resultant total energy separation. In opposition to the results of Hilsch [2] and Hartnett and Eckert [6], Erdélyi [8] postulated that the difference in temperature of the cold and hot air depends only on the type of gas used and the pressure ratio at the inlet and outlet of the vortex tube. In an experimental study Takahama [9] obtained data for the design of a standard type vortex tube with a high efficiency of energy separation. He also gave formulae for the profiles of the velocity and temperature of the air flowing through the vortex tube. Bobrovnikov *et al.* [10] reported on the influence of the humidity of air on the energy separation. Their experiments with different vortex tubes showed that the humidity of air diminishes the effect on the energy separation in a vortex tube. Linderstrøm-Lang [11] studied in detail the application to gas separation, using different gas mixtures and tube geometries. It was indicated that the separation effect depends mainly on the cold and hot gas mass flow ratio. The effect of the gas separation was also confirmed by Marshall [12] by using several different gas mixtures and several different sizes of tubes. He showed that there exists a critical inlet Reynolds number for the maximum effect on the separation.

In the present paper, new experimental results for the energy separation in a vortex tube will be reported. A similarity relation for the prediction of the temperature of the cold exit air, obtained from the dimensional analysis and confirmed by experimental data, will be presented.

2. EXPERIMENTAL APPARATUS

The experimental apparatus used for the determination of the energy separation in a vortex tube is shown schematically in Fig. 1. Compressed air passing through the inlet valve (2) and the filter nozzle (3), is led tangentially into the vortex tube (1). The air is expanded in the vortex tube and divided into a cold and a hot stream. The cold air leaves the central orifice near the entrance nozzle, while the hot air discharges at the periphery at the far end of the tube. The flow rate of the hot air can be controlled by the control valve (10). The mass flow rates of the hot and cold air are determined by measuring the pressure drops across the standard orifice devices (13) and (15) by using two Betz manometers, and the temperatures by using the thermocouples (12) and (14). The temperatures of the inlet air, the cold and hot air leaving the vortex tube, are measured by the thermocouples (5), (6) and (8) respectively. The pressure of the inlet air is measured by the manometer (4). In order to obtain the temperature

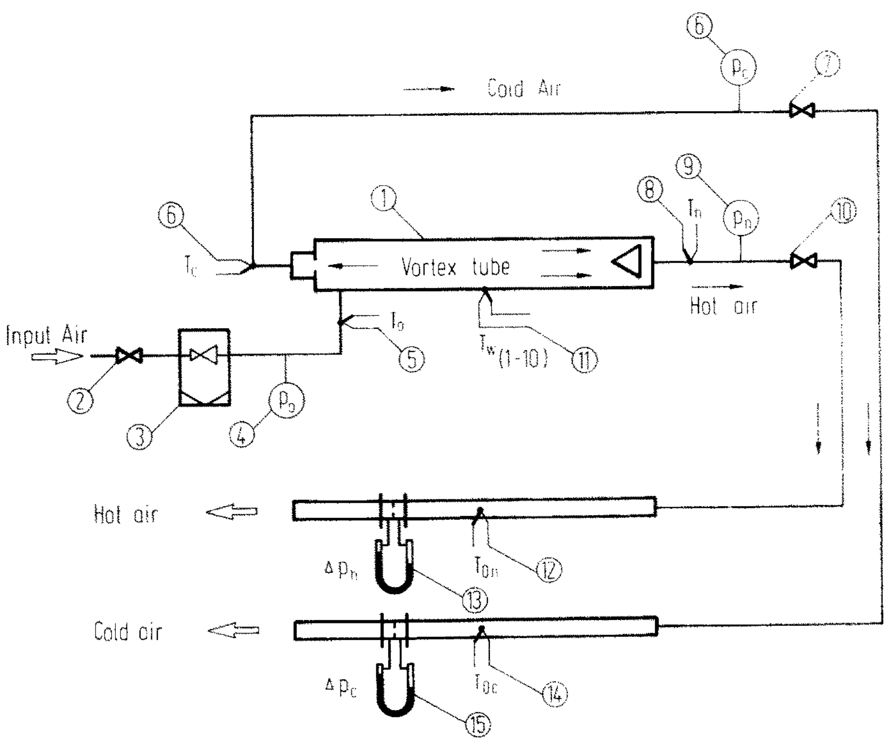

FIG. 1. Schematic diagram of the experimental apparatus.

distribution along the surface of the vortex tube, there are 10 thermocouples (11) distributed at equal intervals of 25 mm on the tube surface as shown in Fig. 2. All the temperatures measured by the thermocouples are recorded by a temperature recorder. The geometrical configuration of the experimental vortex tube is similar to one of the tubes used by Hilsch [2]. It was constructed to give as low a cold gas temperature as possible [13]. The geometrical data of the vortex tube are as follows:

Inner diameter $D = 17.6$ mm,
length $L = 20D = 352$ mm,
diameter of the inlet air nozzle $\delta = 4.1$ mm,
diameter of the cold exit air orifice $d_c = 6.5$ mm.

During the experimental test, the pressure of the inlet air, p_0, and the cold air mass ratio, v, were varied. The cold air mass ratio is defined as follows:

$$y = \dot{m}_c/\dot{m}_0 = \frac{\text{Mass flow rate of the cold air}}{\text{Total air mass flow rate}}. \tag{1}$$

3. EXPERIMENTAL RESULTS

The experimental temperature differences between the hot exit air and the inlet air, $\Delta T_h = T_h - T_0$ and between the inlet air and the cold exit air, $\Delta T_c = T_0 - T_c$, as functions of the cold air mass ratio, $y = \dot{m}_c/\dot{m}_0$, with the pressure of the inlet air as a parameter, are presented by the solid lines in Fig. 3. Without the outflow of the cold air, $y = 0$, the temperature measured at the center of the central orifice used for the exit of the cold air, is lower than the temperature of the inlet air, due to the effect of energy separation. Similarly without the outflow of the hot air, $y = 1$, the temperature of the vortex tube wall at the hot air exit end is higher than that of the inlet air. A sharp temperature drop is measured for the hot air exit temperature near $y = 1$. This result is different from the experimental results of Hilsch [2], in which the temperature of the hot air increases with y, and reaches its highest value at $y = 1$.

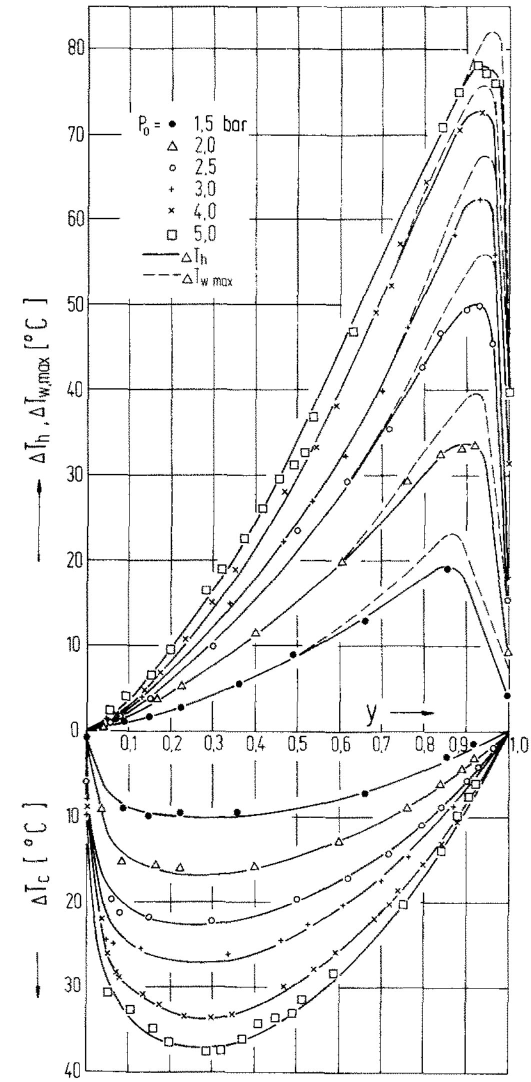

FIG. 3. Temperature differences, ΔT_h and ΔT_c, as functions of the cold air mass ratio, y, with the pressure of the inlet air, p_0, as a parameter.

A possible flow pattern of the counter flow in the vortex tube, proposed by Fulton [3], is shown in Fig. 4. The stream surfaces of revolution are represented by the solid lines. The resulting net outward flow of energy is indicated by the radial arrows. During the energy separation process there exists a stagnation point S on the axis of the vortex tube, and a stream surface through this point divides the future cold air from the future hot air. As indicated by Fulton, slightly downstream toward the hot air exit of this stagnation point, the air has a radial temperature distribution ranging from a

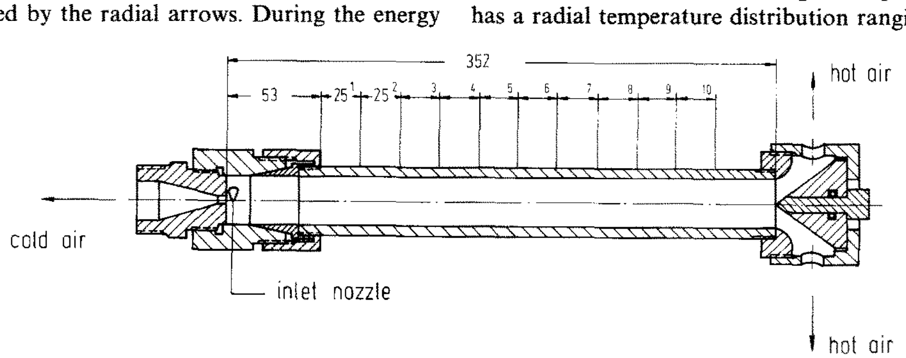

FIG. 2. Location of the 10 thermocouples along the surface of the vortex tube.

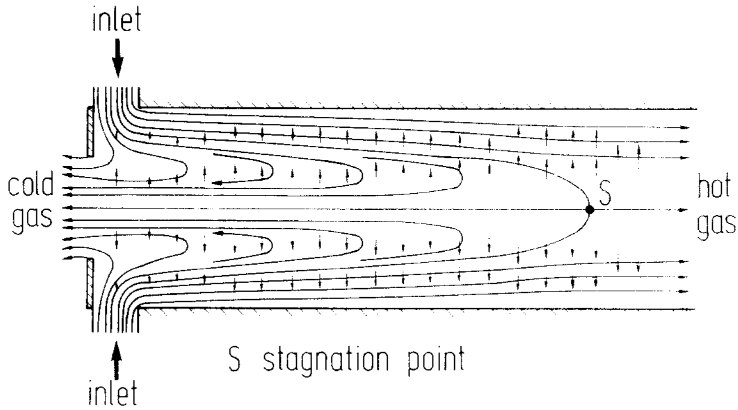

FIG. 4. Supposed flow pattern in a counter flow vortex tube [3].

low value at the axis to a very high one at the wall. Furthermore Fulton stated that the tube wall in that region is hotter than the final mixed air, and hotter than the tube wall either at the inlet or at the far end of the tube. For a high hot air flow rate or a small value of y, the highest temperature of the tube wall is located near the far end of the tube. Therefore, it is expected that the stagnation point on the axis of the vortex tube is also near the far end of the tube. Figure 5 shows the maximum temperature along the vortex tube wall, measured by thermocouples 1–10 located on the tube wall surface as shown in Fig 2. Figure 6 gives the temperature differences between the wall and the inlet temperature, $\Delta T_w = T_w - T_0$, along the vortex tube wall, with y as a parameter, for two inlet pressures $p_0 = 2$ bar and 4 bar. From this figure the following two conclusions can be made:

(1) The temperature of the vortex tube wall increases with an increase of the cold air mass ratio, y, except for $y = 1$.

(2) An increase of the value of y results in a relocation of the maximum temperature to the inlet end of the vortex tube, as shown in Figs. 5 and 6. The variation of the maximum temperature with respect to the variation of y is shown by the dashed lines in Fig. 3.

It is well known that a Görtler vortex appears on the surface, as shown schematically in Fig. 7 [14], when a fluid flows along a concave wall. From the experimental results in Fig. 8, it can be concluded that the Görtler vortex is a major driving force for the energy separation in a vortex tube. This figure shows the experimental results for an inlet pressure of $p_0 = 4$ bar. Curve (a) represents the case of a well insulated vortex tube, and curve (b) represents the case without insulation, where an air stream flows with a velocity of 4 m s^{-1} across the vortex tube. It is seen that the hot air temperature in case (b) is smaller than that in case (a) due to the heat losses to the surroundings. Because of the heat losses from the tube wall, the strength of the Görtler vortex on the inside wall of the tube decreases. This results in a decrease in the driving force for the energy separation. Therefore the cold air temperature

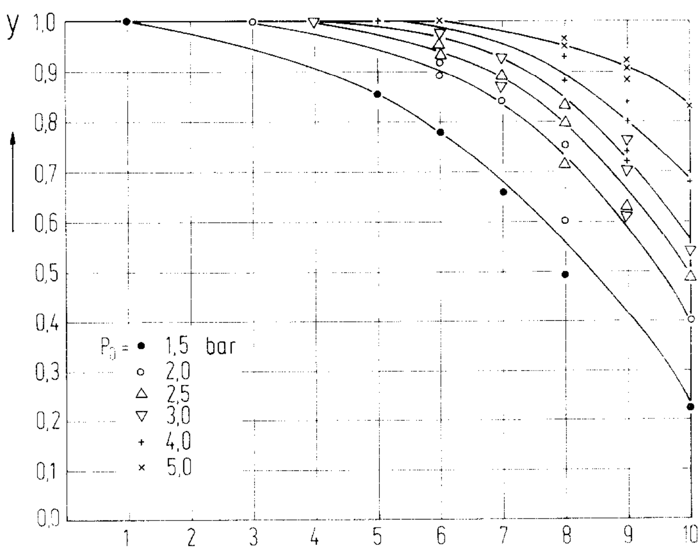

FIG. 5. Maximum temperature along the vortex tube wall, pressure of the inlet air as a parameter. (Numbers on the abscissa indicate the number of thermocouples located on the vortex tube surface as shown in Fig. 2.)

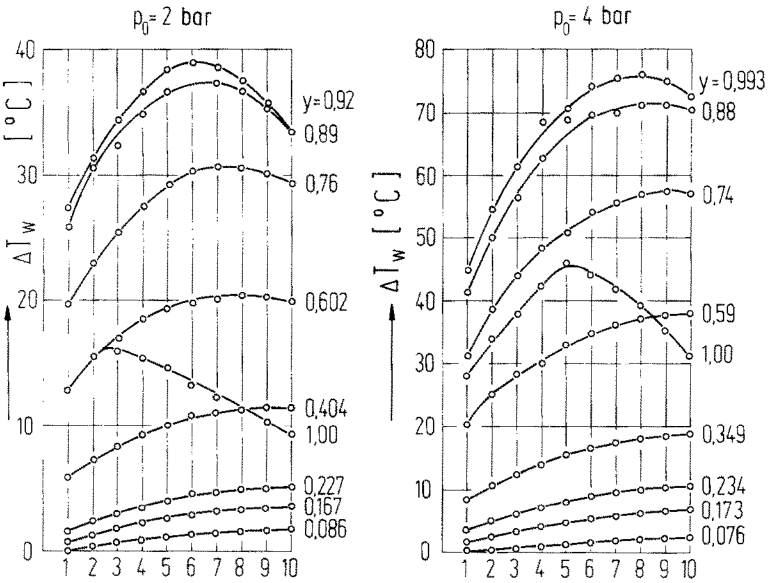

FIG. 6. Temperature along the vortex tube wall with y as a parameter for $p_0 = 2$ and 4 bar. (Numbers on the abscissa indicate the number of thermocouples.)

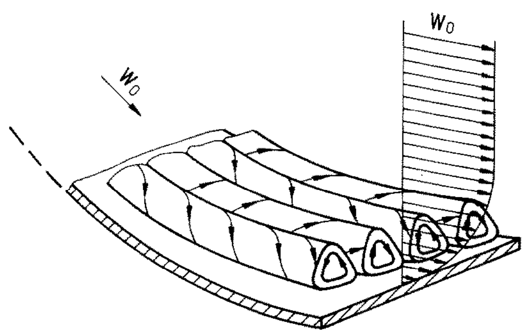

FIG. 7. Schematic diagram of the Görtler vortex on a concave wall.

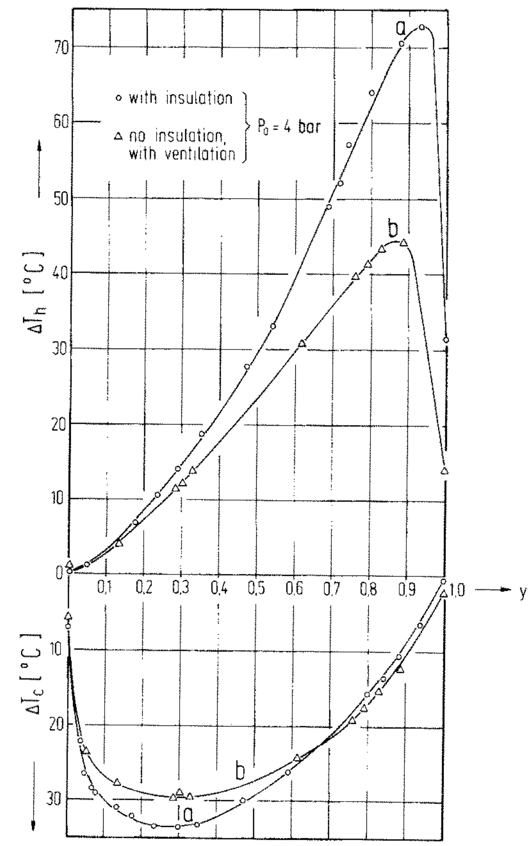

FIG. 8. Temperature differences, ΔT_h and ΔT_c, as functions of the cold air mass ratio, y, for $p_0 = 4$ bar. (a) The vortex tube is well insulated. (b) The vortex tube has no insulation, air stream velocity = 4 m s^{-1}.

increases, especially in the region of the minimum cold temperature. In the region $y > 0.7$, i.e. more than 70% of the total air mass leaves at the cold end, part of the cold air comes from the region near the tube wall where the temperature is higher than that near the axis of the tube. Due to the heat losses from the tube wall this part of the cold air is precooled. Therefore the temperature of the cold air decreases slightly for $y > 0.7$.

4. DIMENSIONAL ANALYSIS

We consider a steady state axisymmetrical compressible flow in a vortex tube according to Fig. 9 with negligible body force and without an energy source. The vortex tube is well insulated from its surroundings. The gas flowing through the vortex tube is considered to be an ideal gas.

For the process of energy separation in the vortex

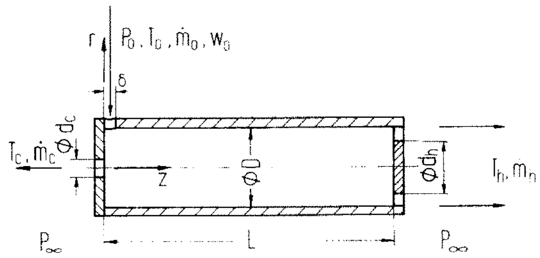

FIG. 9. Schematic diagram of the geometrical configuration of the vortex tube.

tube, the temperature of the cold exit gas is an important variable which remains to be determined. To predict the cold exit gas temperature, the method of dimensional analysis will be used. The independent parameters, which are involved in the energy separation process, can be listed as follows:

The entrance conditions:
- temperature T_0,
- pressure p_0,
- density ρ_0,
- tangential velocity w_0.

The ambient pressure p_∞.

The properties of the gas:
- thermal conductivity k,
- specific heat at constant pressure c_p,
- specific heat at constant volume c_v,
- dynamic viscosity μ,
- coefficient of thermal expansion β.

The operation parameter:
- mass flow rate of the cold gas $\dot{m}_c$.

The geometrical dimensions of the vortex tube:
- length L,
- inner diameter D,
- width of the inlet nozzle δ,
- diameter of the orifice at the cold gas exit d_c,
- diameter of the restriction at the hot gas exit d_h.

For the prediction of the temperature T_c of the cold exit gas, the following functional relationship can be written:

$$f(T_c, T_0, p_0, \rho_0, w_0, p_\infty, k, c_p, c_v, \mu, \beta, \dot{m}_c, L, D, \delta, d_c, d_h) = 0. \quad (2)$$

From the dimensional analysis, the following dimensionless parameters can be obtained:

$$\gamma = c_p/c_v, \quad (3)$$

$$Ma = w_0/(\gamma p_0/\rho_0)^{1/2} \quad (4)$$

$$Re = \rho_0 w_0 D/\mu, \quad (5)$$

$$Pr = c_p \mu/k, \quad (6)$$

$$Eu = p_\infty/(\rho_0 w_0^2), \quad (7)$$

$$C^* = c_p/(p_0/\rho_0 T_0), \quad (8)$$

$$\beta^* = \beta T_0, \quad (9)$$

$$p^* = p_0/p_\infty, \quad (10)$$

$$\theta_c = (T_0 - T_c)/T_0, \quad (11)$$

$$L^* = L/D, \quad (12)$$

$$\Delta = \delta/D, \quad (13)$$

$$D_c = d_c/D, \quad (14)$$

$$D_h = d_h/D, \quad (15)$$

$$y = \dot{m}_c/\dot{m}_0, \quad (16)$$

where

$$\dot{m}_0 = \pi D \delta \rho_0 w_0. \quad (17)$$

Then equation (2) can be expressed in dimensionless form as follows:

$$\theta_c = F(Ma, Re, Pr, Eu, \gamma, C^*, \beta^*, p^*, L^*, \Delta, D_c, D_h, y). \quad (18)$$

As a first approximation, equation (18) can be represented by the following equation for a small variation of the pressure of the inlet air:

$$\theta_c = f_1(Ma) f_2(Re) f_3(Pr) f_4(Eu) f_5(\gamma) \times f_6(C^*) f_7(\beta^*) f_8(p^*) f_9(L^*) f_{10}(\Delta) \times f_{11}(D_c) f_{12}(D_h) f_{13}(y). \quad (19)$$

Experimental results indicate that, when y varies from 0 to 1.0 and the other parameters are kept constant, θ_c possesses a maximum value which can be determined from equation (19) as follows:

$$\theta_{c,\max} = f_1(Ma) f_2(Re) f_3(Pr) f_4(Eu) \times f_5(\gamma) f_6(C^*) f_7(\beta^*) f_8(p^*) f_9(L^*) \times f_{10}(\Delta) f_{11}(D_c) f_{12}(D_h) f_{13}(y_{\max}). \quad (20)$$

Dividing equation (19) by equation (20), we obtain

$$\theta_c/\theta_{c,\max} = \frac{f_{13}(y)}{f_{13}(y_{\max})} = g(y). \quad (21)$$

Substituting the definition for θ_c, equation (11), into equation (21) and letting

$$\Delta T_c = T_0 - T_c \quad (22)$$

and

$$\Delta T_{c,\max} = (T_0 - T_{c,\max}), \quad (23)$$

we obtain

$$\frac{\Delta T_c}{\Delta T_{c,\max}} = g(y). \quad (24)$$

Equation (24) represents a similarity relation for the variation of the cold exit gas temperature as a function of y for different vortex tubes being geometrically similar to each other. Figure 10 shows the similarity relation of equation (24) obtained from the experimental curves for ΔT_c from Fig. 3. It can be seen that all 6 curves for ΔT_c from Fig. 3 can be reduced to a single

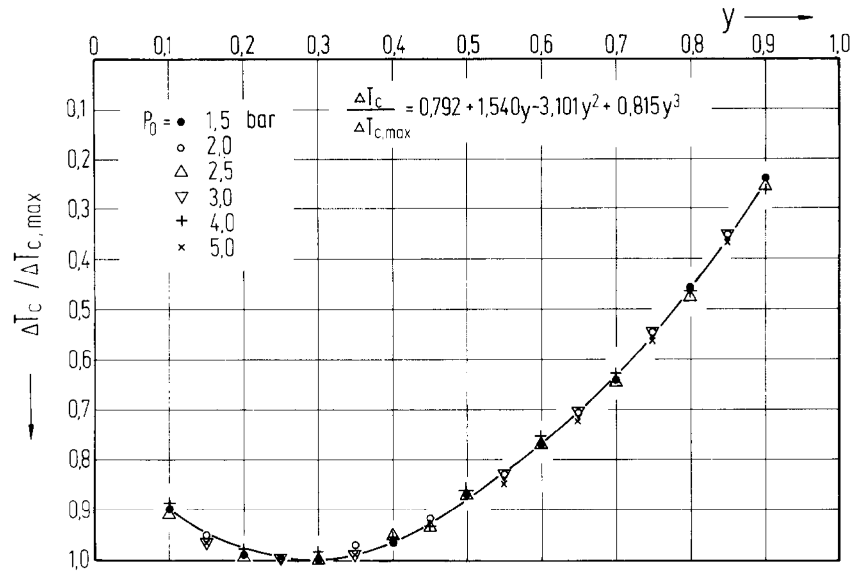

FIG. 10. Similarity relation, equation (25) compared with experimental data from Fig. 3.

curve by the similarity relation. It can be expressed by the equation

$$\frac{\Delta T_c}{\Delta T_{c,max}} = 0.792 + 1.540y - 3.101y^2 + 0.815y^3. \quad (25)$$

Figure 11 shows the similarity relation, $\Delta T_c/\Delta T_{c,max}$ as a function of y, obtained from the experimental results of Hilsch [2]. Except for $p_0 = 10$ atm, the similarity relation is well established.

5. CONCLUSIONS

Experimental results of the temperature of the cold and hot air leaving the vortex tube, with the pressure of the inlet air, p_0, and the cold air mass ratio, y, as parameters, are presented. It is shown that the temperature of the hot exit air drops sharply in the region near $y = 1$. This result differs from previous published data. The temperature and the variation of the location of the maximum temperature along the vortex tube surface provide useful information about the location of the stagnation point of the flow field at the axis of the vortex tube. Experimental results indicate that the Görtler vortex produced by the tangential velocity on the inside wall of the vortex tube is a major driving force for the energy separation in a vortex tube. A similarity relation presented by equation (24), and confirmed by experimental data, can be used for predicting the temperature of the cold air of geometrically similar vortex tubes.

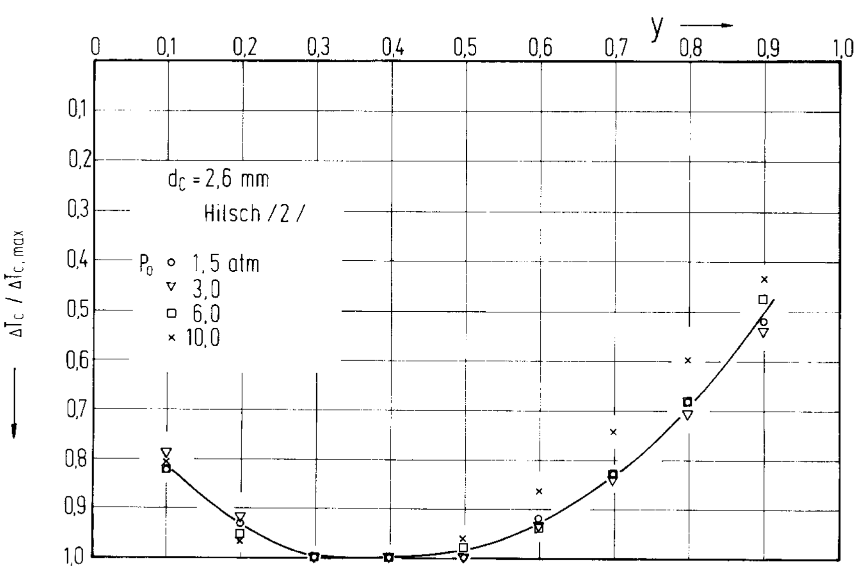

FIG. 11. Similarity relation, equation (25), compared with experimental data of Hilsch [2].

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ETUDE DU PARTAGE D'ENERGIE DANS UN TUBE A VORTEX

Résumé—On étudie en détail le processus de partage d'énergie dans un tube à vortex avec l'air comme fluide. On présente des résultats expérimentaux sur les températures de l'air chaud ou froid qui quitte le tube à vortex. La variation de température pariétale maximale le long de la surface du tube fournit une information utile sur la position du point d'arrêt de l'écoulement sur l'axe du tube. Les résultats expérimentaux montrent que le tourbillon de Görtler produit par la vitesse tangentielle sur la paroi interne du tube est la cause principale de la séparation d'énergie. Une relation de similitude pour la prédiction de la température de sortie de l'air froid, obtenue par l'analyse dimensionnelle, est présentée et confirmée par l'expérience.

UNTERSUCHUNG DER ENERGIEUBERTRAGUNG IM WIRBELROHR

Zusammenfassung—Der Vorgang der Energieübertragung im Wirbelrohr wurde mit Luft als Arbeitsmedium untersucht. Die bei den Versuchen gemessenen Temperaturen der kalten und warmen Luft werden in Form von Diagrammen dargestellt. Aus dem Verlauf der gemessenen maximalen Rohrwandtemperatur längs des Wirbelrohrs für verschiedene Anteile der kalten Luft am Gesamtmassenstrom erhält man Hinweise über die Lage des Staupunktes auf der Achse des Wirbelrohres und über das Strömungsfeld. Die so gewonnenen Meßergebnisse legen den Schluß nahe, daß Görtler Wirbel, die sich aufgrund der tangentialen Strömung an der Wirbelrohrinnenwand bilden, mit als Hauptursache für die Energietrennung genannt werden müssen. Zur Vorausberechnung der Kaltlufttemperatur wurde eine Ähnlichkeitsbeziehung aufgestellt, die über eine Dimensionsanalyse gewonnen wurde. Diese Gleichung gibt die experimentellen Daten sehr gut wieder.

ИССЛЕДОВАНИЕ РАЗДЕЛЕНИЯ ЭНЕРГИИ В ВИХРЕВОЙ ТРУБКЕ

Аннотация—Детально изучен процесс разделения энергии в вихревой трубке, где в качестве рабочей среды используется воздух. Представлены экспериментальные данные по температуре холодного и горячего воздуха, выходящего из вихревой трубки. Изменение максимальной температуры стенки вдоль поверхности вихревой трубки дает полезную информацию о положении точки торможения поля скоростей течения на оси вихревой трубки. Экспериментальные результаты показывают, что вихрь Гертлера, образуемый тангенциальной скоростью на внутренней стенке вихревой трубки, является главной движущей силой разделения энергии. Получено и экспериментально проверено соотношение подобия для расчета температуры холодного выходящего воздуха, полученное анализом размерностей.